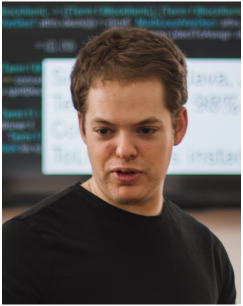
Abstract

In this talk I will show an implementation of angelism non-determinism (such as the ‘amb‘ operator) that uses only mutable state and exceptions. This implementation can be extended to get an implementation of delimited continuations!
It is not necessary to be familiar with non-determinism or continuations to follow the talk. I will start with some introductory background on notions of effects in programming languages: direct vs. indirect style, monads, Filinski’s monadic reflection, and effect handlers.

Tout réussir en répétant beaucoup

James Koppel, **Gabriel Scherer**, Armando Solar-Lezama

June 22, 2018



In one slide

We are going to:

- do something impossible about effects

Something impossible: pure OCaml implementation of *nondeterminism*, which extends to *delimited continuations*.

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- proved correct (by me, in the easy case)

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We are going to:

- do something impossible about effects
- in a disappoingly simple way (Jimmy's neat trick)
- proved correct (by me, in the easy case)
- starting with useful background (for you)

Something impossible: pure OCaml implementation of *nondeterminism*, which extends to *delimited continuations*.

Section 1

Background on effects

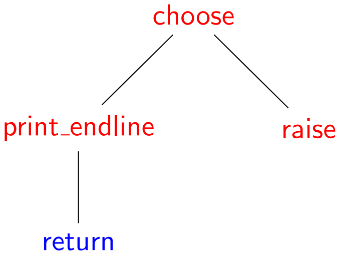
The core of programming:
(de)constructing values + performing function calls.

The rest is *side effects*:

- state
- Input/Output
- exceptions
- non-determinism
- system calls
- continuations
- ...

a computation tree

```
if choose [true; false]
then (print_endline "it worked"; 42)
else raise (Failure "oops")
```



(computation goes down and up again)

Direct and indirect style

```
let rec enum_nqueens i qs =
  if i = n then qs else
    let q = choose (List.filter (okay qs) range) in
    enum_nqueens (i+1) (q :: qs)
```

Direct and indirect style

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let rec enum_nqueens i qs =
  if i = n then qs else
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let rec enum_nqueens i qs =
  if i = n then [qs]
  else List.fold_left
          (fun sols q → if not (okay qs q) then sols
                         else enum_nqueens (i+1) (q :: qs) @ sols)
          [] range
```

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let rec enum_nqueens i qs =
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         [] range

let rec enum_nqueens i qs =
  if i = n then ListMonad.return qs else
    ListMonad.bind (List.filter (okay qs) range) (fun q →
      enum_nqueens (i + 1) (q :: qs)
    )
```

Filinski's monadic reflection (1994)

```
module Reflect (M : Monad) : sig
  val reflect : 'a M.t → 'a
  val reify : (unit → 'a) → 'a M.t
end
```

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module Reflect (M : Monad) : sig
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module Choice = Reflect(ListMonad)

let rec enum_nqueens i qs =
  if i = n then qs else
    let q = Choice.reflect (List.filter (okay qs) range) in
    enum_nqueens (i+1) (q :: qs)

let solutions = Choice.reify (fun () → enum_nqueens 0 [])
```

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let solutions = Choice.reify (fun () → enum_nqueens 0 [])
```

Possible in any language with delimited continuations (shift/reset).

Effect handlers (Plotkin and Pretnar, 2009)

```
effect Choose : 'a list → 'a

let rec enum_nqueens i qs =
  if i = n then qs
  else
    let q = perform (Choose (List.filter (okay qs) range)) in
    enum_nqueens (i + 1) (q :: qs)

let with_choice m =
  match m () with
  | r → [r]
  | effect (Choose li) k →
    List.flatten (List.map (fun v → continue k v) li)

let solutions = with_choice (fun () → enum_nqueens 0 [])
```

(Implemented in Multicore OCaml.)

```
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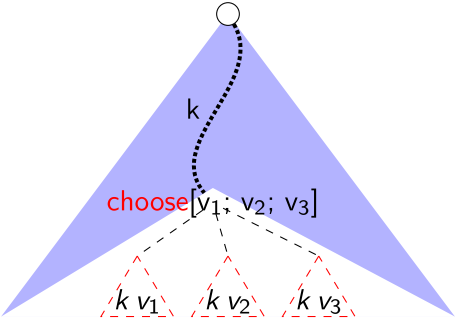
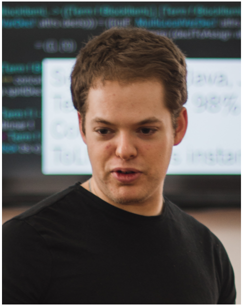


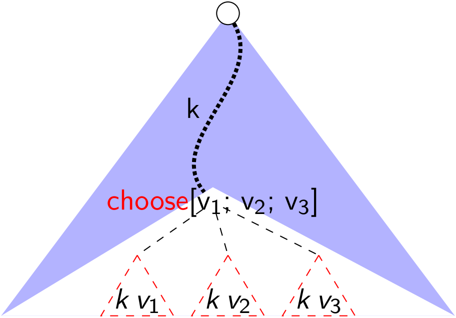
(uses continuations again)

Section 2

Jimmy's neat trick

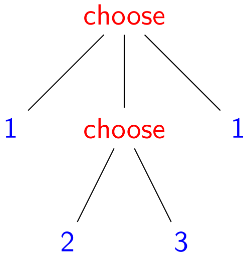


val choose : 'a list → 'a
val with_choice : (unit → 'a) → 'a list



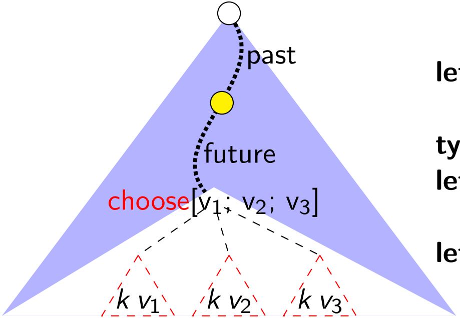
Jimmy's trick: if we can't *capture* k, just *replay* it.

```
with_choice (fun () →
  if choose [true; false; true] then 1
  else
    if choose [true; false] then 2 else 3
)
```



On replay, remember the value

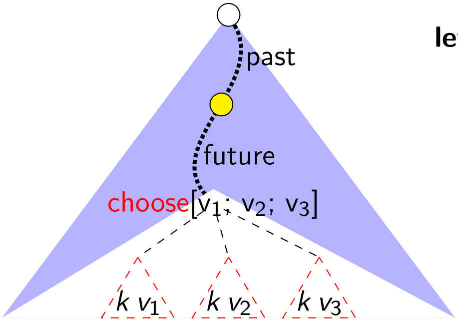
Setup (1/3)




```
type idx = int * int
let start_idx xs = (0, List.length xs)
let next_idx (k, len) =
  if k + 1 = len then None
  else Some (k + 1, len)
let get xs (k, len) = List.nth xs k

type 'a stack = 'a list ref
let push stack x =
  stack := x :: !stack
let pop stack = match !stack with
  | [] → None
  | x::xs → stack := xs; Some x
```

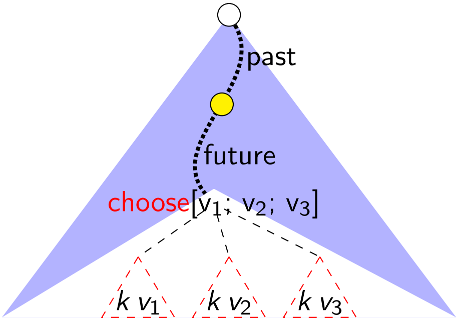
choose (2/3)




```
let past = ref []
let future = ref []
exception Empty

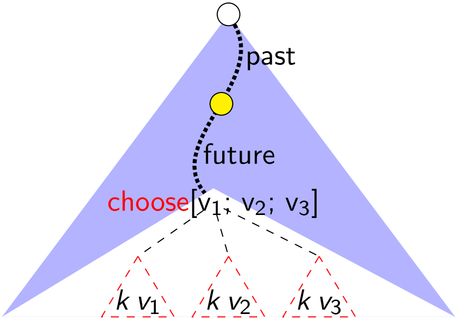
let choose = function
  | [] → raise Empty
  | xs →
    let i = match pop future with
    | None → start_idx xs
    | Some i → i
    in
    push past i;
    get xs i
```

with_choice (3/3)



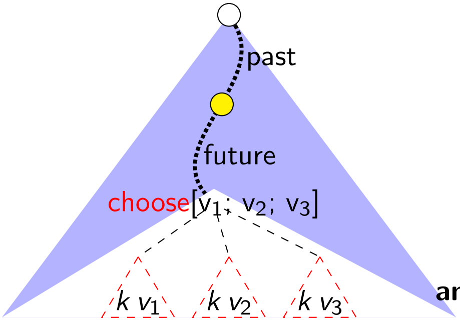

```
let rec with_choice f = loop f []
and loop f acc =
  let r =
    try [f ()] with Empty → [] in
  let acc = r @ acc in
```

with_choice (3/3)




```
let rec with_choice f = loop f []
and loop f acc =
  let r =
    try [f ()] with Empty → [] in
  let acc = r @ acc in
  match next_path !past with
  | None → List.rev acc
  | Some path →
    past := [];
    future := List.rev path;
    loop f acc
```

with_choice (3/3)

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  let acc = r @ acc in
  match next_path !past with
  | None → List.rev acc
  | Some path →
    past := [];
    future := List.rev path;
    loop f acc
and next_path = function
  | [] → None
  | i::is →
    match next_idx i with
    | Some i' → Some (i'::is)
    | None → next_path is
```

Delimited continuations

Jimmy extended this idea to implement *delimited continuations*.

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Implementation:

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- ... yet very hard to understand

Not in this talk!

https://arxiv.org/abs/1710.10385

Section 3

Non-determinism: correctness proof

Continuation machines

$$\boxed{(t, K, s, R)} \qquad (t, \mathsf{halt}, \emptyset, \emptyset)$$

$$
\begin{aligned}
t, u ::= & \\
& \mid x, y, z \ldots \\
& \mid n \in \mathbb{N} \\
& \mid \mathsf{S}\ t \\
& \mid \mathsf{let}\ x = t\ \mathsf{in}\ t' \\
& \mid \mathsf{choose}\ x\ y
\end{aligned}
\qquad
\begin{aligned}
K ::= & \\
& \mid \mathsf{S}\ K \\
& \mid \mathsf{let}\ x = \square\ \mathsf{in}\ (t, K) \\
& \mid \mathsf{halt}
\end{aligned}
$$

$$
\begin{aligned}
s &::= \emptyset \mid (t, K).s \\
R &::= \emptyset \mid n.R
\end{aligned}
$$

Continuation machines

$$\boxed{(t, K, s, R)} \qquad (t, \mathsf{halt}, \emptyset, \emptyset)$$

$$\begin{array}{lcl} (\mathsf{S}\ t, K, s, R) & \rightarrow & (t, \mathsf{S}\ K, s, R) \\ (n, \mathsf{S}\ K, s, R) & \rightarrow & (n+1, K, s, R) \end{array}$$

Continuation machines

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(n, \mathsf{S}\ K, s, R) & \rightarrow & (n+1, K, s, R) \\
(\mathsf{let}\ x\ =\ t\ \mathsf{in}\ t', K, s, R) & \rightarrow & (t, (\mathsf{let}\ x\ =\ \Box\ \mathsf{in}\ (t', K)), s, R)
\end{array}
$$

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(\mathsf{S}\ t, K, s, R) & \to & (t, \mathsf{S}\ K, s, R) \\
(n, \mathsf{S}\ K, s, R) & \to & (n+1, K, s, R) \\
(\mathsf{let}\ x = t\ \mathsf{in}\ t', K, s, R) & \to & (t, (\mathsf{let}\ x = \Box\ \mathsf{in}\ (t', K)), s, R) \\
(n, \mathsf{let}\ x = \Box\ \mathsf{in}\ (t', K), s, R) & \to & (t'[x \leftarrow n], K, s, R)
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\end{array}$$

$$(\mathsf{choose}\ n_1\ n_2, K, s, R) \quad \to \quad (n_1, K, (n_2, K).s, R)$$

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\end{array}
$$

$$
\begin{array}{lcl}
(\mathsf{choose}\ n_1\ n_2, K, s, R) & \rightarrow & (n_1, K, (n_2, K).s, R) \\
(n, \mathsf{halt}, (n', K).s, R) & \rightarrow & (n', K, s, n.R)
\end{array}
$$

History machines

$$\boxed{(t, K, P, F, R)_u} \qquad (t, \mathsf{halt}, \emptyset, \emptyset, \emptyset)_t$$

$$i \quad ::= \quad 1 \mid 2 \qquad \begin{array}{lcl} P & ::= & \emptyset \mid P.i \\ F & ::= & \emptyset \mid i.F \end{array}$$

History machines

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$$\begin{aligned}
(\mathsf{S}\ t, K, P, F, R)_u &\to (t, \mathsf{S}\ K, P, F, R)_u \\
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(\mathsf{let}\ x\ =\ t\ \mathsf{in}\ t', K, P, F, R)_u &\to (t, \mathsf{let}\ x\ =\ \square\ \mathsf{in}\ (t', K), P, F, R)_u \\
(n, \mathsf{let}\ x\ =\ \square\ \mathsf{in}\ (t', K), P, F, R)_u &\to (t'[x \leftarrow n], K, P, F, R)_u
\end{aligned}$$

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(n, \mathsf{let}\ x\ =\ \square\ \mathsf{in}\ (t', K), P, F, R)_u & \rightarrow (t'[x \leftarrow n], K, P, F, R)_u
\end{array}$$

$$(\mathsf{choose}\ n_1\ n_2, K, P, \emptyset, R)_u \quad \rightarrow \quad (\mathsf{choose}\ n_1\ n_2, K, P, 1.\emptyset, R)_u$$

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(\mathsf{choose}\ n_1\ n_2, K, P, \emptyset, R)_u & \rightarrow & (\mathsf{choose}\ n_1\ n_2, K, P, 1.\emptyset, R)_u \\
(\mathsf{choose}\ n_1\ n_2, K, P, (i.F), R)_u & \rightarrow & (n_i, K, (P.i), F, R)_u
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(n, \mathsf{halt}, P, \emptyset, R)_u & \to & (u, \mathsf{halt}, \emptyset, P{+}1, n.R)_u
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\end{array}
$$

$$
\begin{array}{lcl}
P.1{+}1 & \stackrel{\text{def}}{=} & P.2 \\
P.2{+}1 & \stackrel{\text{def}}{=} & P{+}1
\end{array}
$$

Proof: combined machines

$$\boxed{(t, K_P, F, s, R)_u} \qquad (t, \mathsf{halt}_\emptyset, \emptyset, \emptyset, \emptyset)_t$$

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$$\begin{array}{lcl}
(\text{choose } n_1\ n_2, K_P, \emptyset, s, R)_u & \rightarrow & (n_1, K_{P.1}, \emptyset, (n_2, K_{P.2}).s, R)_u \\
(\text{choose } n_1\ n_2, K_P, i.F, s, R)_u & \rightarrow & (n_i, K_{P.i}, F, s, R)_u \\
(n, \mathsf{halt}_P, \emptyset, (n', K_{P'}).s, R)_u & \rightarrow & (n', K_{P'}, \emptyset, s, n.R)_u
\end{array}$$

Proof: combined machines

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\end{array}$$

Proof: timeline and replay

$$(n, \mathsf{halt}, P, \emptyset, R)_u \rightarrow (u, \mathsf{halt}, \emptyset, P{+}1, n.R)_u$$
$$(n, \mathsf{halt}_P, \emptyset, (n', K_{P'}).s, R)_u \rightarrow (n', K_{P'}, \emptyset, s, n.R)_u$$

$$(n, \mathsf{halt}_P, \emptyset, (n', K_{P'}).s, R)_u \rightarrow (u, \mathsf{halt}_\emptyset, P', s, n.R)_u \rightarrow^* (n', K_{P'}, \emptyset, s, n.R)_u$$

Proof: timeline and replay

$$(n, \mathsf{halt}, P, \emptyset, R)_u \rightarrow (u, \mathsf{halt}, \emptyset, P{+}1, n.R)_u$$
$$(n, \mathsf{halt}_P, \emptyset, (n', K_{P'}).s, R)_u \rightarrow (n', K_{P'}, \emptyset, s, n.R)_u$$

$$(n, \mathsf{halt}_P, \emptyset, (n', K_{P'}).s, R)_u \rightarrow (u, \mathsf{halt}_\emptyset, P', s, n.R)_u \rightarrow^* (n', K_{P'}, \emptyset, s, n.R)_u$$

Timeline Invariant:

$$P' = P{+}1$$

$$(\mathsf{choose}\ n_1\ n_2, K_P, \emptyset, s, R)_u \rightarrow (n_1, K_{P.1}, \emptyset, (n_2, K_{P.2}).s, R)_u$$

Proof: timeline and replay

$$\begin{array}{lcl}(n, \mathsf{halt}, P, \emptyset, R)_u & \rightarrow & (u, \mathsf{halt}, \emptyset, P{+}1, n.R)_u \\ (n, \mathsf{halt}_P, \emptyset, (n', K_{P'}).s, R)_u & \rightarrow & (n', K_{P'}, \emptyset, s, n.R)_u\end{array}$$

$$(n, \mathsf{halt}_P, \emptyset, (n', K_{P'}).s, R)_u \rightarrow (u, \mathsf{halt}_\emptyset, P', s, n.R)_u \rightarrow^* (n', K_{P'}, \emptyset, s, n.R)_u$$

Timeline Invariant:

$$P' = P{+}1$$

$$(\mathsf{choose}\ n_1\ n_2, K_P, \emptyset, s, R)_u \quad \rightarrow \quad (n_1, K_{P.1}, \emptyset, (n_2, K_{P.2}).s, R)_u$$

Replay Theorem:

$$\mathsf{replay}(n, K_P, F, s, R)_u \stackrel{\mathsf{def}}{=} (u, \mathsf{halt}_\emptyset, (P.F), s, R)_u$$

$$(t, \mathsf{halt}_\emptyset, \emptyset, \emptyset)_t \rightarrow^* c \quad \Longrightarrow \quad \mathsf{replay}(c) \rightarrow^*_{\mathsf{pure}} c$$

(Witty transition slide)

Section 4

Benchmarks!

Worst case is very bad

```
with_choice (fun () →
  let v = long_pure_computation () in
  let i = choose [0; 1; 2; 3; 4; 5; 6; 7; 8; 9] in
  (i, v)
)
```

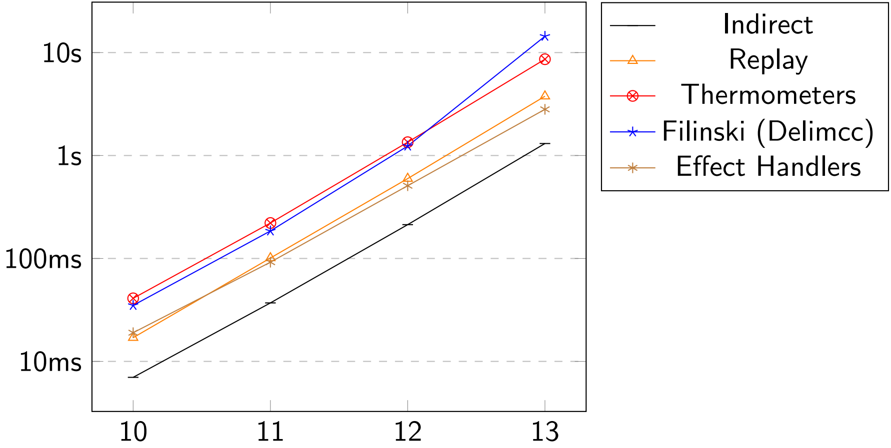
N queens

```
let n = int_of_string Sys.argv.(1)
let range = List.init n (fun i → i)

let okay qs q =
  let rec okay i c = function
    | [] → true
    | x::xs →
      c <> x && (c−x) <> i && (c−x) <> −i && okay (i+1) c xs
  in okay 1 q qs

let rec enum_nqueens i qs =
  if i = n then qs else
    let q = choose (List.filter (okay qs) range) in
    enum_nqueens (i+1) (q :: qs)

let nb_sols = List.length (with_choice (fun () → enum_queens 0 []))
```



	10	**11**	**12**	**13**
Indirect	0.007s	0.037s	0.213s	1.308s
Replay	0.017s	0.101s	0.597s	3.768s
Therm.	0.041s	0.221s	1.347s	8.621s
Filinski (Delimcc)	0.035s	0.185s	1.236s	14.412s
Effect Handlers (Multicore OCaml)	0.019s	0.092s	0.509s	2.81s
Prolog search (GNU Prolog)	0.165s	0.614s	3.307s	20.401s

Thanks. Any questions?

```
queens(N, N, L, L).
queens(N, I, L, Res) :-
  I < N,
  choose_okay_in_range(0, N, C, L),
  I1 is I+1,
  queens(N, I1, [C|L], Res).

choose_okay_in_range(I, N, I, L) :- I < N, okay(1, I, L).
choose_okay_in_range(I, N, C, L) :-
  I < N, I1 is I+1, choose_okay_in_range(I1, N, C, L).

okay(_, _, []).
okay(I, C, [X|XS]) :-
  C =\= X, (C-X) =\= I, (X-C) =\= I, I1 is I+1, okay(I1, C, XS).

count(N, Count) :- aggregate_all(count, queens(N, 0, [], L), Count).
```